Computational studies of Insect-sized Flapping Wings in Inclined stroke plane under the influence of temporally varying shear inflow

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Abstract

2D computational studies on the effects of temporally varying shear inflow condition on forces generated by an insect-sized flapping wing in the inclined stroke plane have been carried out for Re = 150. Temporally varying shear inflow condition was represented by a combination of a mean wind speed term, a temporally fluctuating velocity term and a spatial velocity gradient term. The mathematical form followed the expression

\[ \frac{U_G}{U_w} = \frac{U_G}{U_w} \sin \left( \frac{2\pi f_G t}{V_{grad}} \right) + \left( \frac{U_G}{U_w} \right) \frac{V_{grad}}{U_w} \frac{y}{U_w}. \]

Here, \( U_G \) is the gusty inflow velocity in m/s, \( U_\infty \) is mean free stream velocity in m/s, \( U_g \) is the amplitude of the temporally sinusoidal velocity fluctuation in m/s, \( V_{grad} \) is the linear velocity gradient along the Y-axis in m/s per m, \( U_w \) is the root mean square average of the flapping cycle velocity of the wing in m/s, \( f_g \) is the frequency of the temporally sinusoidal velocity fluctuation in Hz, \( f_w \) is the wing’s flapping frequency in Hz, \( y \) is dimension along Y-axis in m and \( t \) is time in second. For the present studies \( U_g/U_w=1 \) & \( f_g/f_w=0.1 \) were considered. Five cases with linear velocity gradient ratio along y-axis, \( V_{grad}/U_w=\pm 5, \pm 2.5 \) & 0 were considered. Findings were analyzed by plots of instantaneous and cycle-averaged force coefficients, phase space plots, global recurrence plots and windowed recurrence quantification analysis. Numerical investigations revealed that negative \( V_{grad}/U_w \) induced a considerable increase in vertical forces and marginally decreased the horizontal forces. Positive \( V_{grad}/U_w \) induced a meagre increase in horizontal forces but caused a substantial decrement in vertical forces.

Keywords: MAV, NAV, flapping wings, inclined stroke plane, temporally varying shear inflow, global recurrence plots, windowed recurrence quantification analysis

1 Introduction

Flapping wing Micro and Nano Aerial Vehicles (MAVs / NAVs) are popular areas of research, both in academic and industrial circles, across India [1 – 3] as well as the globe. Various aspects like aerodynamics of insect-sized flapping wing under different modes of flight (hover, ascending, descending, forward flight & manoeuvres), flow physics of single & tandem wings with different wing-to-wing positions and kinematics, aero-elastic studies of flexible flapping wings and corrugated wings have been extensively studied by researchers. Ground effects on the flow field in the environs of the flapping wings and swarm dynamics have also been deliberated. Reviews of literature covering these aspects have been periodically reported [4 – 7].

Researchers like Lian & Shyy[8], Wan & Huang [9], Viswanath & Tafti [10], Prater & Lian [11], Sarkar et al [12], Zhu et al. [13] and Jones and Yamaaleev [14] studied effects of unsteady inflow conditions on the flapping wing. However, most of them have dealt with inflow conditions represented by a temporally varying sinusoidal function or a Heaviside function. However, a real-life inflow condition has wind shear also. Keeping in mind this aspects, the present study focused on investigating insect-sized wings flapping in inclined stroke plane subjected to a temporally varying shear inflow profile. The shear velocity inflow gradient was varied and their effects on the force patterns were computationally studied. Re = 150 was considered for the present simulation. Flapping was along inclined stroke plane and pitching of the wing was restricted during the pronation and supination phases of flapping cycle. Combination of a non-zero mean inflow velocity, a sinusoidally fluctuating temporal term and a 1D shear gradient term was considered to model temporally varying shear inflow condition. The shear gradient term was varied and its effect on force patterns was numerically studied.
2 Numerical Formulation, Wing Kinematics & Domain Architecture

2D unsteady Navier-Stokes equations were solved using finite volume formulation, assuming incompressible and laminar flow at Re = 150. Mass and momentum equations were solved in a fixed inertial reference frame by the Arbitrary Lagrangian-Eulerian (ALE) formulation. The generic integral form of the conservation equation for a scalar quantity $\varphi$ in an arbitrary control volume, $V$, with moving boundaries inside it is shown in Eq. 2.1.

$$\frac{d}{dt}\int_V \rho \varphi \,dV + \int_{\partial V} \rho \varphi (\vec{u} - \vec{u}_g) \cdot d\vec{A} = \int_{\partial V} \Gamma \Delta \varphi \cdot d\vec{A} + \int_V \mathbf{S}_\varphi \,dV$$

(2.1)

Here $\rho$ is the fluid density, $\vec{u}$ is the flow velocity, $\vec{u}_g$ is the velocity of the moving mesh, $\Gamma$ is the diffusion coefficient and $\mathbf{S}_\varphi$ is the source term.

Spatial discretization was second order upwind and the time discretization was second order implicit. PISO scheme was used for pressure-velocity coupling. Convergence of the iterative method was considered to be satisfied when mass and momentum residues decreased below $O(10^{-6})$ in magnitude. Finite Volume formulation based commercial CFD code ANSYS Fluent was used to solve the 2D time-dependent unsteady Navier-Stokes equations.

Wing kinematics and parameters considered were the same as prescribed by Sudhakar and Vengadesan\textsuperscript{[15]}. Variations of $x$ and $y$ coordinate of the wing mid-point and the pitching angle are shown in Fig. 1. Wing rotation was confined to 20% of the flapping cycle during pronation and supination phases. Details of the domain and boundary conditions were the same as reported in De et al\textsuperscript{[16]}. The wing was represented by a flat plate with $t_{max}/c$ ratio of 2%.

**Figure 1 Wing Kinematics**

3 Methodology

Inclined stroke plane wing kinematics documented by Sudhakar and Vengadesan\textsuperscript{[15]} was chosen. UDF for simulating the prescribed kinematics of the wing in 2D reference frame was developed. Domain, grid and time step independency studies were carried out to arrive at a reasonable domain size, grid resolution and time step. Validation studies were carried out for benchmark simulations documented by Sudhakar & Vengadesan\textsuperscript{[15]} at Re = 150. Findings of the validation studies are shown in Fig. 2.

Subsequently, Users Defined Function (UDF) for specifying temporally varying shear inflow conditions was developed. The expression is shown in Eq 3.1.

$$\frac{U_G}{U_w} = \frac{U_w}{U_w} + \left(\frac{U_w}{U_w}\right) \sin \left(2\pi \frac{f_g}{f_w} t\right) \pm \left(V_{\text{grad}} \frac{U_w}{U_w}\right) y$$

(3.1)

Here, $U_G$ is the total inflow velocity in m/s, $U_w$ is mean free stream velocity in m/s, $U_g$ is the amplitude of the temporally sinusoidal velocity fluctuation in m/s, $V_{\text{grad}}$ is the linear velocity gradient along the Y-axis in m/s per m, $U_w$ is the root mean square average of the flapping cycle velocity of the wing in m/s, $f_g$ is the frequency of the temporally sinusoidal velocity fluctuation in Hz, $f_w$ is the wing’s flapping frequency in Hz, $y$ is dimension along Y-axis in m and $t$ is time in second. For our present study, $U_G/U_w=1$ & $f_g/f_w=0.1$ were considered. These were chosen based on our previous studies\textsuperscript{[16]} on a series of $U_G/U_w$ & $f_g/f_w$. Five cases with linear velocity gradient ratio along y-axis, $V_{\text{grad}}/U_w = \pm 5$, $\pm 2.5$ & $0$, were considered.

Findings were compared in terms of instantaneous as well as gust cycle averaged non-dimensionalized vertical & horizontal force coefficients, phase diagrams of vertical to horizontal force coefficients, global recurrence plots (GRPs) and windowed recurrence quantification analysis (WRQA).
4 Results & Discussion

Findings of the present studies are reported in the following sub-sections.

4.1 Force coefficients

Instantaneous horizontal and vertical force coefficient histories are shown in Figs. 3(a) and (b) respectively.

Figure 3 (a) Temporal variation of instantaneous vertical force coefficient

Figure 3 (b) Temporal variation of instantaneous horizontal force coefficient

Gust cycle averaged vertical and horizontal force coefficients are shown in figure 4 (a) and (b) respectively.

Figure 4 (a) Average vertical force coefficient

Figure 4 (b) Average horizontal force coefficient

4.2 Phase Plots

Phase plots of vertical to horizontal force coefficients for $V_{\text{grad}/U_w} = \pm 5, \pm 2.5 & \text{0}$ are plotted and shown in Figs. 5 (a) - (e).

Figure 5 Phase Plots
4.3 Global recurrence plots (GRPs)

GRP is an X-Y plot of the times at which a phase space trajectory visits roughly the same area in the phase space at a given moment in time. They describe the distances of every point $x(i)$ to all the other points $x(j)$ in the phase space diagram. The plot is represented by the below mathematical expression:

$$G(i, j) = \rho(||x(i) - x(j)||)$$

(4.3.1)

Here, $i$ & $j = 1, 2, 3, \ldots, N$, $\|\cdot\|$ is the Euclidean norm operator and $\rho(\cdot)$ is the color code operator that maps the distance to a color scale. GRPs of vertical and horizontal force coefficients for $V_{\text{grad}}/U_w = \pm 5, \pm 2.5 & 0$ were plotted and are shown in Figs. 6 (a) & (b) respectively.

4.4 Windowed recurrence quantification analysis (WRQA)

WRQA helped to quantitatively assess any event and precisely judge the possibility of an onset of a system’s unstable behaviour. In the present study, all the variables reported in Webber and Marwan [17] were employed. They are recurrence rate (RR), determinism (DET), laminarity (LAM), trapping time (TT), ratio (RATIO), entropy (ENTR), maximum line ($L_{\text{MAX}}$) and trend (TREND). In the present study, the time period of one flapping cycle was considered as one window period width. This choice was based on the fact that gust caused variation in the inflow conditions between two consecutive flapping cycles. It was envisaged to assess that whether due to the difference in the inflow conditions, the recurrence variables exhibited uneven fluctuations for a given pair of frequency and velocity ratios. A noticeable uneven fluctuation in the recurrence variable series would indicate an onset of instability. Standard deviations of these recurrence variable series were calculated to ascertain the uneven fluctuation of the series. The threshold for computing the recurrence variables was chosen as 5 times the standard deviation to avoid the effect of noise in the signals. All variables were calculated for a dimension of 1. Euclidean norm was used while defining the neighbourhood radius.

4.4.1 Recurrence Rate (RR)

Recurrence Rate is the measure of the density of recurrence points. It is computed as below:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}^{m} x 100$$

(4.4.1.1)

Here $N$ is the length of data series and $R_{i,j}^{m}$ is the recurrence matrix of an m-dimensional phase space trajectory and a neighbourhoods radius $\varepsilon$. In the present case, $m$ was chosen as 1 and $\varepsilon$ was chosen in such a manner that most of the RRs in the WRQA were approximately 1 % [17]. RR series for force coefficients are shown in Figs. 7 (a) & (b).
4.4.2 Determinism (DET)

Determinism (aka predictability) is the ratio of recurrence points that form a diagonal line of length $l_{min}$ to all the recurrence points. For systems with stochastic nature, DET has smaller value and for systems with periodic behaviour, DET has larger values. It is computed as below:

$$DET = \frac{\sum_{i=1}^{N} P^\varepsilon(l)}{\sum_{j=1}^{N} P^\varepsilon(l)} \times 100$$

(4.4.2.1)

Here $P^\varepsilon(l)$ is the frequency distribution of the lengths $l$ of the diagonal structures in the recurrence plot with a neighborhoods radius $\varepsilon$ and $l_{min}$ is the minimum threshold diagonal line. For the present studies, $l_{min} = 2$ was considered. DET series for force coefficients are shown in Figs. 8 (a) & (b).

4.4.3 Laminarity (LAM)

Laminarity is the ratio of recurrence points that form a vertical line of length $v_{min}$ to all the recurrence points. It is the measure of the number of vertical structures in the whole recurrence plot and represents the occurrence of a laminar state in the system without describing the length of these laminar phases. LAM generally decrease if the occurrence of single recurrence points is more than the vertical structures. Mathematical expression to compute LAM is as below:

$$LAM = \frac{\sum_{v=1}^{N} v P^\varepsilon(v)}{\sum_{l=1}^{N} P^\varepsilon(l)} \times 100$$

(4.4.3.1)

Here $P^\varepsilon(v)$ is the frequency distribution of the lengths $v$ of the vertical structures in the recurrence plot with a neighborhoods radius $\varepsilon$ and $v_{min}$ is the minimum threshold vertical line. For the present studies, $v_{min} = 2$ was considered. LAM series for force coefficients are shown in Figs. 9 (a) & (b).

4.4.4 Trapping Time (TT)

Trapping Time is computed from the vertical structures of the recurrence plots. It represents information about the amount and length of the vertical structures in the recurrence plots. It measures the meantime that the system will be trapped in a specific state. TT is computed as below:

$$TT = \frac{\sum_{v=v_{min}}^{N} v P^\varepsilon(v)}{\sum_{l=v_{min}}^{N} P^\varepsilon(v)}$$

(4.4.4.1)

TT series for force coefficients coefficient are shown in Figs. 10 (a) & (b).
Ratio (RATIO)

Ratio is defined as the ratio between DET and RR. It is used to discover transitions in the behaviour of a dynamic system subjected to the fact that during certain types of transitions RR decreases, whereas DET does not change at the same time. RATIO is computed as below:

\[
RATIO = \frac{N \sum_{i=l_{\text{min}}}^{N} p^*(i)}{\left(\sum_{i=l_{\text{min}}}^{N} p^*(i)^2\right)^{1/2}}
\]  

(4.4.5.1)

RATIO series for force coefficients are shown in Figs. 11 (a) & (b).

Entropy (ENTR)

Entropy refers to the Shannon entropy of the frequency distribution of diagonal line length. It reflects the complexity of the deterministic structures in a dynamic system. ENTR is calculated as below:

\[
ENTR = -\sum_{l=l_{\text{min}}}^{N} p(l) \log p(l) = \frac{\sum_{l=l_{\text{min}}}^{N} p^*(i)}{\left(\sum_{i=l_{\text{min}}}^{N} p^*(i)^2\right)^{1/2}}
\]  

(4.4.6.1)

Here, \(l_{\text{min}} = 2\) was considered. ENTR series for force coefficients are shown in Figs. 12 (a) & (b).

Maximum Line (L_MAX)

This variable deals with the divergence of the trajectory segment. It gives information about the range in which a segment of the trajectory is close to one another at a different time. Length of the diagonal lines is related to the largest positive Lyapunov exponent and characterizes the rate of separation of the infinitesimally closed trajectories in the phase space diagram. L_MAX is computed using the below expression:

\[
L_{\text{max}} = \max \left( \left\{ l_i \mid i=1, \ldots, N \right\} \right)
\]  

(4.4.7.1)

L_MAX series for force coefficients are shown in Figs. 13 (a) & (b).
4.4.8 Trend (TRENDS)

Trend is a linear regression coefficient over the recurrence point density of the diagonal lines parallel to the Line of Identity (LoI). It is a function of the time distance between diagonals and LoI. It gives information about the non-stationary nature in the process like a drift. It is computed using the below expression:

\[
\text{TRENDS} = \frac{\sum_{i=1}^{N} (i-N/2) \cdot (RR_i-(RR_{0}))}{\sum_{i=1}^{N} (i-N/2)^2}
\] (4.4.8.1)

Here, \((RR_i)\) is the average Recurrence rate of the \(i^{th}\) diagonal line. TRENDS series for force coefficients are shown in Figs. 14 (a) \& (b).

From these studies, it was observed that negative \(V_{\text{grad}}/U_w\) induced a considerable increase in the gust cycle averaged vertical force \& meagre decrease in gust cycle averaged horizontal force. Positive \(V_{\text{grad}}/U_w\) gradient induced a marginal increase in gust cycle averaged horizontal force but a substantial decrease in gust cycle averaged vertical force. GRP patterns of the force coefficients exhibited a checkerboard patterns for all the shear gradient inflow conditions. This indicated that their variation was periodic in nature and was a superposition of harmonic oscillations. The secondary checkerboard patterns that were observed apart from the primary patterns were indications of presence of periodic temporal change in the force pattern due to \(f_\gamma\). WRQA series for all the eight parameters quantitatively indicated that the fluctuation of the windowed series varied with the shear gradient. For the vertical force coefficients, the fluctuations increased as the shear gradient varied from
negative to positive value. For horizontal force coefficients, the fluctuations increased as the shear gradient varied from positive to negative value.

5 Conclusion

Finding of the numerically investigate insect-sized flapping wings, i.e. Re of the order $O(10^2)$, in inclined stoke plane under the influence of temporally varying shear inflow condition have been reported in this paper. Effect of positive and negative shear inflow profile on force patterns was assessed. From the application point of view, this know-how would lead to improvement of the existing control algorithms for kinematics of flapping wings and hence improve their performance. For example, if the anthropogenic insect-sized flyer experiences a flow field with spatial variation of velocity, the inclined-stroke plane angle can be modulated such that a negative $V_{\text{grad}}/U_w$ gradient is maintained, thus improving the lift force pattern. Similarly, when the need of higher thrust arises, the inclined-stroke plane angle can be modulated to maintain a positive $V_{\text{grad}}/U_w$ gradient. This would help to the realization of MAVs & NAVs with better flight stability & longer flight endurance.

References: